

2/6/18 "The strength to win comes from within."-Anonymous

HW: "Horizontal Stretching of functions" homework section

Test 1 on Thursday 2/15

AIM: How do we recognize a horizontal stretch?

Warm Up:

The function $h(x)$ has a range given by the interval $[-2, 8]$. The function $f(x)$ is defined by $f(x) = \frac{1}{2}h(x) + 6$.

What is the range of $f(x)$?

$$\begin{array}{r}
 -2 \leq y \leq 8 \\
 \cdot \frac{1}{2} \qquad \cdot \frac{1}{2} \\
 \hline
 -1 \leq y \leq 4 \\
 +6 \qquad \qquad +6 \\
 \hline
 5 \leq y \leq 10
 \end{array}$$

or

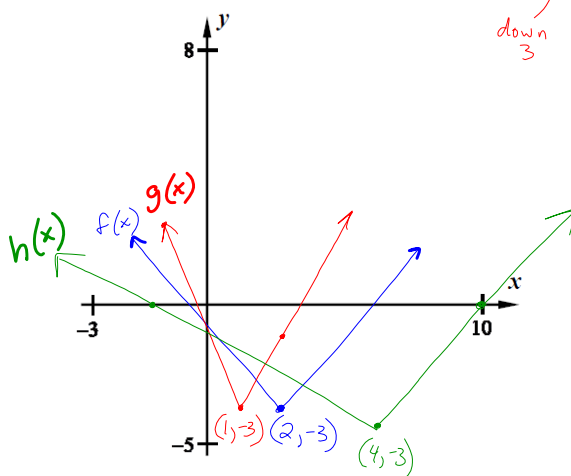
$$[5, 10]$$

①
Vertical
compression
times y
by $\frac{1}{2}$

②
Shift up
6 units
add 6
to y

Exercise #1: Consider the absolute value function $f(x) = |x-2| - 3$.

- (a) Using your calculator, sketch a graph of f on the axes provided. Label the coordinates of its vertex point without the use of your calculator.



- (b) Consider the function $g(x) = f(2x)$. Determine a formula for g and then graph it on the axes. Use your calculator to find its minimum point and label it on the graph.

$$f(x) = |x-2| - 3$$

$$g(x) = |2x-2| - 3$$

- (c) Now consider the function $h(x) = f\left(\frac{1}{2}x\right)$. Determine a formula for h and graph it on the axes. Use your calculator to find its minimum point and label it on the graph.

$$f(x) = |x-2| - 3$$

$$h(x) = \left|\frac{1}{2}x - 2\right| - 3$$

- (d) Summarize your findings below for each function.

$f(x)$ turning point:

$$(2, -3)$$

$f(2x)$ turning point:

$$(1, -3)$$

$f\left(\frac{1}{2}x\right)$ turning point:

$$(4, -3)$$

- (e) What stayed constant about the turning points? What changed and how did it change?

Y-value
Stayed
the same

The x-value changed
by doing the inverse
of what was shown.

HORIZONTAL DILATIONS

For a real number, positive constant such that $k > 1$:

1. The function $f(kx)$ represents a horizontal compression of $f(x)$ by a factor of k
2. The function $f\left(\frac{1}{k}x\right)$ represents a horizontal stretch of $f(x)$ by a factor of k .

Exercise #2: Let's take a look at the quadratic function $f(x) = x^2 - 12x + 20$.

- (a) Determine the coordinates of its turning point by using the equation for the axis of symmetry of

$$x = -\frac{b}{2a}$$

$$b = -12$$

$$a = 1$$

$$x = \frac{-(-12)}{2(1)} = 6$$

$$f(6) = 6^2 - 12(6) + 20 = -16$$

$$(6, -16)$$

- (b) If g is defined by $g(x) = f(3x)$, what should be the coordinates of its turning point based on our previous work? Explain.

horizontal compression

$$6 \div 3 = 2$$

(Divide x-value by 3)

$$(2, -16)$$

- (c) Determine a formula for $g(x)$ and then use the turning point formula to verify your answer from part (b).

$$g(x) = f(3x)$$

$$g(x) = (3x)^2 - 12(3x) + 20$$

$$g(x) = \underset{a}{9}x^2 - \underset{b}{36}x + \underset{c}{20}$$

$$x = \frac{-b}{2a}$$

$$x = \frac{-(-36)}{2(9)} = \frac{36}{18} = 2$$

$$g(2) = 9(2)^2 - 36(2) + 20 = -16$$

Turning Pt

$$(2, -16) \checkmark$$

- (d) Show that the y-intercept of both $f(x)$ and $g(x)$ are equal. What does this make sense from a horizontal dilation perspective?

$$x = 0$$

$$f(0) = 0^2 - 12(0) + 20 = 20$$

$$g(0) = 9(0)^2 - 36(0) + 20 = 20$$